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THE VALIDITY OF THE SCHWARZSCHILD RELATION AS APPLIED TO THE USE OF THE LOGARITHMIC SECTOR

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ABSTRACT. A direct experimental test is made of the Schwarzschild relation as applied to the logarithmic sector used in conjunction with a quartz spectrograph. The relation is found to hold within the limits of the experiment, from which it follows that the length of a spectral line on the photographic plate is proportional to the logarithm of its intensity.

1. *Introduction*

A GREAT deal of work has been done upon the reciprocity law, and upon the validity of the Schwarzschild expression, which latter states that for a definite blackening the product It^p is constant (I being the intensity which acts for the time t to give this stated blackening, and p being the Schwarzschild constant). There still remains, however, a considerable diversity of opinion as to whether the above expression always holds for continuous exposures, whilst extremely conflicting views are found in the literature regarding intermittent exposures. At the moment, then, the only correct position to take up is that stated by Jones, Huse and Hall (1) who say "Unless it has been definitely determined that the Schwarzschild relation holds to within the required precision for the particular photographic material and range of intensities used, no reliance should be placed on its validity."

The work to be described constitutes a direct experimental test of the Schwarzschild relation as applied to the logarithmic sector used in conjunction with a quartz

spectrograph. The sector has already been described in these *Transactions* (2), and consists of a disc the periphery of which forms a curve given by

$$-\log\theta = 0{\cdot}3 + 0{\cdot}2l \qquad \ldots\ldots(1),$$

where l is the distance in millimetres measured radially inwards from the outermost part of the curve, and θ is the circumferential aperture. The effect of rotating such a disc in front of a spectrograph slit is to give an exposure whose duration varies along the slit, and the exposure t at any point distant l from the circumference of the disc is obviously directly proportional to the aperture θ. We can hence rewrite (1) as

$$-\log t = 0{\cdot}2l + C \qquad \ldots\ldots(2),$$

where C is a constant depending only on the shape of the sector and its speed of rotation.

For a stigmatic instrument, such as the quartz spectrograph employed in this work, the effect of this variation in exposure along the slit is to produce on the photographic plate lines which are wedge-shaped, and whose lengths are greater the greater their intensity, and in the use of the sector the lengths of the lines are used as indications of their intensities. If we assume that the Schwarzschild relation holds for the wedge sector so used, and substitute in (2) the value for t which is obtainable from the Schwarzschild expression, then we obtain the relation

$$\log I = 0{\cdot}2pl + k \qquad \ldots\ldots(3),$$

where k is a constant, I being the intensity of a particular line, and l being the length of that line on the photographic plate.

If the intensity be varied, as in the following case, merely by altering the distance (d) of the source from the spectrograph slit, then

$$I \propto \frac{1}{d^2} \qquad \ldots\ldots(4),$$

and hence

$$-\log d = 0{\cdot}1pl + A \qquad \ldots\ldots(5),$$

A being a constant.

It follows that if—in a set-up consisting of a source giving a line spectrum, a logarithmic sector, and a quartz spectrograph—all the conditions of exposure remain unchanged except the distance of the source from the spectrograph slit, then, *if the Schwarzschild relation holds in this particular case*, the length of a line (l) on the photographic plate must vary with the distance (d) of the source as indicated in equation (5). In other words, plotting l against $\log d$ should give a straight line and the slope of this line will be $-p/10$.

2. *Experimental*

As a source we employed a condensed spark between tin electrodes. Since no alternating current was available it was necessary to employ a $\frac{1}{4}$ kW. rotary converter running off 220 V. D.C., and this delivered 150 V. A.C., at 50 cycles. This was stepped up by means of a $\frac{1}{4}$ kW. transformer capable of giving 15,000 V. on open

circuit*. A condenser of capacity 0·005 microfarad was placed in parallel with the spark.

Chisel-ended electrodes, set with their edges parallel to the optical axis of the system, were used in order to prevent errors due to the spark wandering laterally between the electrodes. These chisel-ends were cut afresh before every exposure in order to render conditions during the various exposures as uniform as possible. Care was also taken to ensure that the trace of the spark was always completely within the aperture of the spectrograph, thus all the light passing the slit reached the plate. This is essential in order that the inverse square law may be applied. A spark gap of 4 mm. was employed.

The spectrograph employed was the E 3 by Messrs Adam Hilger, Ltd., whilst the logarithmic sector was also made by this firm. The particular sector employed rotated about 300 times per minute at full speed and, after winding up, would run at this speed for just over 3 minutes. We ran the sector at full speed and used exposures of 2 minutes; the sector was fully wound up after every exposure and allowed to run for at least 5 seconds (in order to allow of it acquiring a uniform speed) before the exposure was commenced by starting the spark.

In the examination of a particular brand of plate five, six or seven exposures of 2 minutes each were made on one plate with the source at distances ranging from 60 cm. to 3 m. The plates were developed by means of Rodinal, and the Watkins "Time-Temperature" method was employed.

Measurement of the lines was effected by means of a × 12 eyepiece fitted with a 15 mm. scale divided into tenths of a millimetre. Five independent readings were made of the length of each line, and these were averaged to give a final value. On certain plates complete sets of readings were made by two independent observers, and it was found that the means obtained by the two observers agreed to within 0·1 mm. for good lines, whilst more than 75 per cent. of the lines gave agreement to within 0·2 mm. An error of 0·1 mm. in the measurement of the length of a line is equivalent to an error of $4\frac{1}{2}$ per cent. in the intensity of that line, whilst 0·2 mm. is equivalent to an intensity error of 9 per cent. These percentage errors in the intensity are independent of the absolute intensity of the line.

3. *Results*

The results are given in full for one plate only. In Table I are given the details —for this particular plate—of the measurements on the various spectral lines employed, whilst these same results are shown graphically in Fig. 1. In general ten lines were examined on each plate, and the length of each line was observed with the source at six different distances. A graph is constructed for each line and so every line employed yields a value for p.

The final values of p obtained in each case for the different plates employed are shown in Table II.

* And without the condenser.

Table I. *Variation of line length (l, mm.) with distance (d) of source. Plate: Wellington Anti-Screen.*

d (cm.)	$\log_{10} d$	λ 3784	λ 3656	λ 3551	λ 3142	λ 3072	λ 2914	λ 2788	λ 2780	λ 2666	λ 2572
60	1·78	11·55	9·6	9·4	9·75	9·95	10·6	10·25	11·3	9·85	11·2
83	1·92	9·9	8·25	7·45	7·95	7·9	9·75	8·25	9·5	7·85	10·05
114	2·06	7·3	6·1	5·95	6·5	6·35	7·9	7·4	8·55	6·75	8·25
158	2·20	5·95	4·6	4·3	5·2	5·1	5·9	5·75	6·3	5·2	6·9
218	2·34	5·05	3·3	2·8	4·2	3·6	4·95	4·6	5·85	3·75	6·15
300	2·48	3·2	2·2	1·85	2·3	2·05	2·65	2·3	3·4	2·65	4·6
p =		0·91	0·96	0·95	0·91	0·93	0·95	0·95	0·98	0·93	0·94

Mean value of $p = 0{\cdot}94$.

4. *Discussion of Results*

The details given in Table I, and the graphs of Fig. 1, show—within the limits of error of this experiment—that the relation developed earlier

$$-\log d = 0{\cdot}1pl + k \qquad \text{.....(5)}$$

is correct. In all the cases examined a straight line gives a good fit of the data. Occasionally, however, it is to be noted that the very short lines (namely, lines of low intensity) deviated farthest from the straight line drawn to represent the results. This deviation is always in the direction of the length of line being greater than we should anticipate. It has never been observed with the fast plates (H. and D. 400–450) employed in this investigation, but only with the slow plates (H. and D. 100), and the effect is not always observed with these. This deviation is possibly to be related to the fact that for fast plates γ (i.e. the slope of the straight-line portion of the H. and D. curve) is independent of the intensity employed; whereas for slow plates this is not the case. Jones, Huse and Hall(8) concluded from their work that the Schwarzschild law would not hold over any considerable intensity range with slow plates. However, our range of intensity is very small compared with that employed in their investigation, and it is probable that the deviation we have observed is merely the first sign of the effect that they studied.

The range of intensities throughout which we have verified the relation given by equation (5) is of interest. In altering the distance of the source from 60 cm. to 300 cm. an intensity range of 25:1 is obtained, whilst lines whose original length (i.e. with the source at 60 cm.) varied from 12·5 mm. to 8·2 mm. were employed. Lines of such lengths have an intensity ratio of approximately 6:1; hence it follows that the verification extends over a range of intensities of 150:1.

An examination has been made of all the plates obtained in this investigation in an attempt to detect whether there is any variation of the Schwarzschild constant with the wave-length of the light employed; there is, however, only the slightest suggestion that p increases as the wave-length diminishes. It has hence been thought justifiable to average out all the values obtained (i.e. with different wave-lengths) for any one particular plate. It is believed that the mean values given in Table II represent the Schwarzschild constant, under the conditions of this experiment, to some 3 per cent.

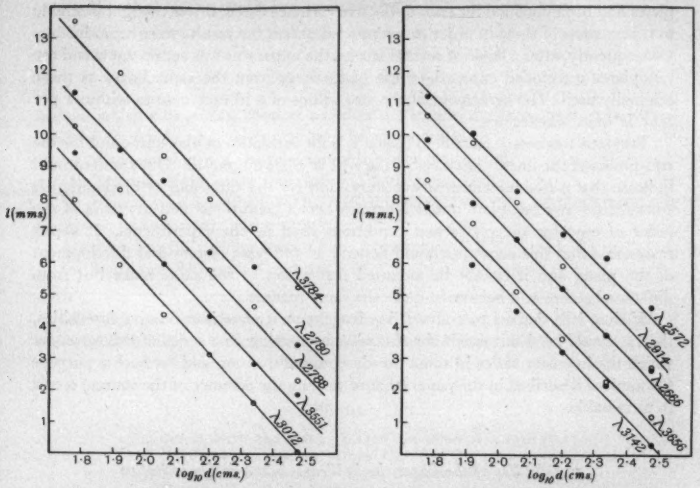

Fig. 1. Graphs showing variation of length of line (l, mm.) with the logarithm of the distance of the source (d, cm.) for the spectral lines given in Table I. In order to avoid confusion the data have been separated into two sets and, in certain cases, the l values of Table I have had an arbitrary amount added—by this means superposition of the graphs is avoided. The alterations are as follows: λ 3784, + 2 mm.; λ 3656, −1 mm.; λ 3142, −2 mm.; λ 3072, −2 mm.

Table II.

Name of plate	Mean value of p	H. and D. number
Wellington Anti-Screen*	0·94	450
" "	0·95	450
" Spectrum Panchromatic	0·95	450
" " (Process)	0·94	100
Ilford Rapid Process Panchromatic	0·97	100
" " "	0·98	100
" Special Rapid Panchromatic	0·95	400

It will be noticed that in Table II two values are listed for both the Ilford Rapid Process Panchromatic and the Wellington Anti-Screen plates†. After exposed

* Details given in full in Table I.

† In view of the work of Twyman and Hitchen, *Proc. Roy. Soc.* **133**A, 1931 (72), it was thought advisable to examine a plate developed in the manner they describe; accordingly, a further Ilford Rapid Process Panchromatic plate was exposed and developed in caustic hydroquinone as used by them. The value of p obtained in this case was 0·93, and the remarks made in the paper *re* the general characteristics of all the plates obtained hold equally well in this particular case.

plates had been obtained for each of the five varieties listed, it was thought desirable to repeat some of these in order to see to what extent the results were reproducible. Consequently, after a lapse of several weeks, the apparatus was set up anew and the two plates mentioned exposed—these plates were from the same boxes as those originally used. The agreement of the two values of p in each case to within 1 per cent. is quite satisfactory.

Previous workers (3) have held that $p = 1$ for intermittent exposures and for the end-points of the lines, regardless of the type of plate employed. Our results would indicate that p (for intermittent exposures and for the threshold of blackening) is constant for any one plate under a definite set of conditions, and that it is of the order of 0·95 for the plates and conditions used in our experiments. It seems reasonable that this constant should depend on the type, history and development of the plate, and it cannot be assumed that plates of the same make but from different batches will behave in quite the same manner.

If then it is desired to convert line lengths on a plate into relative intensities, then p should be determined* *for that plate* by putting on a series of exposures for which the intensity varies in some pre-determined manner, and for such a purpose the method described in the paper (that of varying the distance of the source) seems to us suitable.

5. *The Use of the Logarithmic Wedge Sector in Quantitative Spectrum Analysis*

In the work described an important factor in determining the accuracy of our results has been the constancy, or otherwise, of the spark. In the application of the sector to quantitative spectrum analysis any variation of the spark does not affect the accuracy of the result to nearly such an extent, since in the latter case we are always comparing the intensities of two lines and observing the manner in which this ratio varies. Now, as shown by Twyman and Simeon (2), the ratio of two intensities I_A and I_B is given by

$$\log \frac{I_A}{I_B} = 0{\cdot}2p\ (l_A - l_B) \qquad \text{.....(6),}$$

In the case of comparing intensities then, as is necessary in the quantitative work referred to, all that concerns us is the difference in the lengths of the two lines, and not the actual lengths themselves. If there happen to be slight variations in the spark then we should expect both lines to be affected similarly, and, whilst their lengths might vary, it is reasonable to expect their difference to remain constant; and this is found in practice to be the case. Vagaries of the spark are thus of minor importance in the application of the logarithmic sector to quantitative spectrum analysis.

* The percentage alterations in the intensity estimates, caused by errors in the determination of p, are independent of the length of the line under consideration. An error of 0·01 in p causes a change in the intensity estimates of 2 per cent.

Finally, the relation developed earlier—that

$$\log I = 0{\cdot}2pl + k \qquad \text{.....(3),}$$

or its other form

$$\log \frac{I_A}{I_B} = 0{\cdot}2p\ (l_A - l_B) \qquad \text{.....(6),}$$

both of which expressions connect together the length of a line and its intensity—holds under conditions which may thus be summarised:

(*a*) The use of any of the photographic plates mentioned.

(*b*) The use of the spark and electrical outfit described.

(*c*) The use as described of the optical system consisting of the logarithmic wedge sector and quartz spectrograph.

(*d*) Line lengths varying from 2 mm. to 12 mm.

(*e*) Exposures of length 2 minutes.

We wish to acknowledge the assistance of Mr F. A. Gregory of the technical staff of Adam Hilger, Ltd., in the performance of the experiments.

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(2) Twyman and Simeon. *Trans. Opt. Soc.* **31**, 1929–30 (169).
(3) Holst and Hamburger. *K. Acad. Amst.* **20**, 1918 (1021).
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DISCUSSION

Mr J. Guild: The result obtained by the authors that, under the conditions of these experiments, the Schwarzschild constant p differs by 5 per cent. or thereabouts from unity, is disturbing. The conditions would appear to be essentially the same as those for which Gibson, Howe and others have established that p can be taken as unity with a high degree of accuracy. It has been taken as proved for many years past that if the time from beginning to end of the exposures is the same, and the degree of blackening is the same, the intensities can be compared on the assumption that the effective transmission of a sector is proportional to the sector opening to within the errors of experiment. The present paper appears to contradict this result. It would be interesting to know if this is due to the fact that the measurements here correspond to zero degree of blackening, and it would be useful if similar measurements could be made for points of constant density some distance from the vanishing points of the lines.

Prof. L. C. Martin: When the whole aperture of the collimator of a spectrograph is not filled with light, there is usually some uncertainty as to the distribution of

light into the line image, which may differ appreciably from the geometrical image of the slit and possibly varies with the fully illuminated aperture under any given conditions. It may vary in these experiments, for example, with the variation of the distance of the spark from the slit, and I think it should be made clear what, if any, effect on the results might be expected. Was the slit used in this work wide enough to have an image sensibly uniform across its width? I should like to express my appreciation and thanks to the authors for a piece of work which should make the use of these sector devices much more easy and certain than it has hitherto seemed.

Messrs Twyman and Harvey: The determinations of p given in the present paper are for the threshold of blackening. Some values obtained for points of measured density some distance from the vanishing points of the lines are given in the paper by Twyman and Simeon*; it is hoped to publish further measurements of the same kind in the near future.

Within the range of distances of spark from slit used in these experiments we found no difference in the sharpness of the lines. A slit width of 0·02 mm. was employed throughout.

* *Trans. Opt. Soc.* **31**, 1929–30 (169).

THE "SPEKKER" PHOTOMETER FOR ULTRA-VIOLET SPECTROPHOTOMETRY

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ABSTRACT. The design and construction of the "Spekker" photometer are described, special attention being paid to certain pitfalls of spectrophotometry which have been avoided in this instrument.

Introduction

The first attempt at quantitative ultra-violet spectrophotometry was that of Pflüger (1). He found that the condensed spark between metal electrodes provided a source of radiation sufficiently strong and steady to enable measurements to be made by means of a Rubens thermopile and a galvanometer. This is, in fact, the direct method of measurement still used in the infra-red region.

A little later Houstoun and Anderson (2) secured quantitative measurements in the ultra-violet by means of two ground quartz plates, which were placed between the slit of a spectrograph and a source of ultra-violet radiation. The plates, which acted (owing to the radiations scattered by them) as secondary sources of illumination, were placed at different distances from the light source, the one nearer thereto, therefore, acting as the more intense source and in this way it was possible to compensate the absorption for any wave-length of a substance placed between the slit of the spectrograph and the quartz plate which was nearer to the light source. The method was satisfactory except that the amount of light available was small.

The next serious attempt was that of Victor Henri (3). On the same photographic plate, a series of exposures was made alternately through a layer of solvent of given length and with constant exposure, and through layers of the solution of varying thickness and with adjusted exposure. Points of equal blackening in contiguous strips indicated wave-lengths at which the difference of absorption of solvent and solution was compensated by the varying exposure.

The first spectrophotometer to be manufactured for sale was the Hilger sector photometer. This became available about 1913, and the first published description, except that in the Hilger catalogue, was given by Howe (4). It is this form of spectrophotometer which has been most widely used up to the present time.

A form of polarisation instrument has also been employed, while in the Hilger laboratory a wire grid rotated about an axis parallel to the wires, an arrangement which cuts off radiation uniformly throughout the spectrum in an easily calculable way, has been used with success.

Finally, mention may be made of the Judd Lewis(5) sector photometer, in which two beams of light are admitted through two apertures limited by sectors in the form of quadrants which, by rotating about their bisectors, cut off greater or less amounts of the radiation.

It is to the Judd Lewis photometer that I am indebted for the rhomb system embodied in the "Spekker" photometer, which instrument indeed may rightly be regarded as a greatly simplified form of the Judd Lewis instrument in which the points of optical design have received careful attention.

In all these arrangements, with the exception of Pflüger's, the main principle is the same, that is, the absorption of a substance in one beam is compensated for by reduction either of the amount or duration of the radiation in the other beam.

Photo-electric Cells and Selenium Cells

Of recent years both photo-electric and selenium cells (with or without valve amplification) have been applied to spectrophotometry, and there is no doubt that in appropriate instances increased sensitiveness of measurement is obtainable.

Such devices materially increase the complexity of the apparatus and of the technique. Furthermore, while bringing in fresh sources of error, they do not avoid the errors of the optical apparatus to which they are adjuncts. Such errors are those of wave-length calibration, those due to stray energy, or imperfect alignment of apparatus and others resulting from the introduction of the sample into the beam. The use of a photo-electric cell in place of the eye or of a photographic record diminishes the likelihood of such errors being recognised, with the result that there is a liability that copious and apparently highly accurate results may be amassed which are actually unreliable. A warning against accepting optimistic claims for such photometers has recently been given by Gibson(6). He says:

"The new photo-electric spectrophotometers must stand the same tests for accuracy as any instrument. If the data can be proved accurate, well and good; if not, the user must decide how far the advantages listed above make up for questionable accuracy or demonstrated inaccuracy. Not all laboratories require the degree of accuracy desired, for example, at the national standardising laboratories. There are various problems in control work where accuracy may be of secondary importance. This must be decided by the user of the instrument. He should realise, however, that the curve or value obtained may not be reliable, even though the instrument will duplicate it any number of times at a high rate of speed."

It is, therefore, to be recommended that the addition of the photo-electric cell arrangements, with their peculiar sources of errors and difficulties of technique, should only be resorted to by those who have made themselves perfectly familiar with the well-tried methods of spectrophotometry, the conditions for the successful use of which can be precisely and simply prescribed and assured.

Even where a light sensitive cell is used for ultra-violet spectrophotometry, the photometric part of the arrangement usually follows the same general principle, namely, absorption of the substance placed in one beam is compensated for by cutting down the radiation in the other beam.

Disadvantages of Intermittency

With the first sector photometer issued by Adam Hilger, Ltd., a glass disc was supplied together with its absorption curve in the ultra-violet. This was to enable a calibration of the sector to be made on each photographic plate and thus to avoid errors due to any assumption concerning the Schwarzschild constant. When Howe(4) investigated the Hilger sector method, however, he concluded that the plate integrated intermittent exposures in such a way that comparison of intensities could be made directly in terms of sector openings. His findings were later confirmed by a group of workers at the Bureau of Standards(7) who compared it with visual and photo-electric methods, and still more recently by Baly, Morton and Riding(8) who compared it with the Judd Lewis photometer and also with von Halban's photo-electric values. Although the original rotating sector has thus

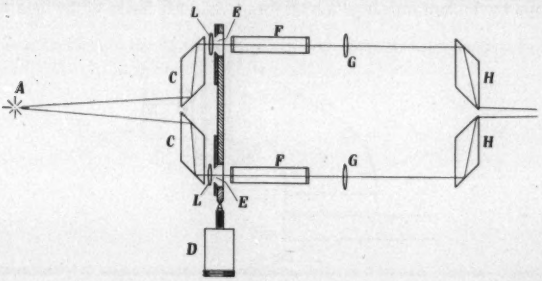


Fig. 1.

borne well the criticisms levelled against it on the score of intermittency, there are other real objections to its use, for instance:

(1) Necessity of stopping the sector to change the adjustment, unless mechanism be introduced which is both complicated and also difficult to make accurately.

(2) Reduction of intensity of each beam to one-half by the sector disc.

(3) The difficulty of alignment. This is not due to the sector disc itself, but to the particular form of optical arrangement adopted for separating the two beams.

(4) Closeness of the scale at high densities.

The "Spekker" Photometer

The photometer which is now made by Adam Hilger, Ltd., under the trade mark "Spekker" is shown in diagram in Fig. 1. It is intended to be placed in front of a spectrograph, such as a quartz spectrograph. Light from a suitable source of ultra-violet rays *A* passes towards the inner edges of two quartz rhombs *C*, *C*, from which beams are diverted upwards and downwards, to be then reflected forward through the tubes *F*, *F*, in one of which is placed the absorbing liquid, and in the

other the tubes of non-absorbing liquid or whatever other substance may be required to be compared—for instance, in the one tube may be an organic substance in a solvent, and in the other tube the solvent alone—the usual device of course for eliminating the reflection of the end plates. The beams from the rhombs pass the one through a fixed rectangular aperture, and the other through an aperture which is variable by a micrometer screw. The screw is rotated by a drum D with an index which travels along a helix in the drum, which not only simplifies readings, but acts as a stop to prevent the drum being turned too far in either direction.

After passing through the absorbing substance and the non-absorbing substances, the beams pass through lenses G, G of quartz, whose focal length is such that an image of the light source is formed on the face of the slit of the spectrograph. A second pair of quartz rhombs H, H are so arranged to bring the beams of light together on to the slit in such a way that the image falling from the top rhomb and that from the bottom rhomb on the slit form a complete image of the light source.

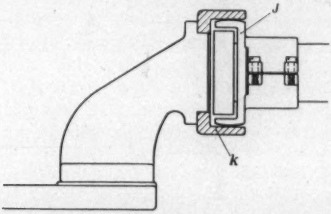


Fig. 2.

The instrument is made and adjusted with considerable care directed to the exact fulfilment of this condition, which has as its object that no matter how much the light source may fluctuate in intensity and position, the portions of the image near where the division by the quartz rhomb takes place will always originate from closely adjacent parts of the light source, and therefore be illuminated with equal intensity.

The lenses G, G are so designed that in addition to forming an image of the light source on the slit of the spectrograph each forms an image on the prism of the spectrograph of the adjustable and fixed diaphragms E, E.

The method of securing and maintaining alignment is indicated in Fig. 2. Rigidly fixed to the slit of the spectrograph is a spherical socket J over which fits a cylindrical fitting K, which cylindrical fitting forms a detachable part of the photometer.

The light source, which forms part of the instrument, once having been adjusted, the photometer fitting K is inserted into the socket, and the photometer pushed towards the slit until it is arrested by two thin rubber strips abutting against the face of the slit, and nothing more requires to be done but to adjust the levelling screws until the observer, looking from the camera end of the spectrograph, sees,

when the instrument is actuated, images of the diaphragms *E*, *E* in the middle of the prism aperture.

Attention must be paid to the width of slit. The real image formed within the spectrograph by the lens is widened by diffraction when the slit width is very small. It is found that no error from this cause arises if, when using the photometer with the small (E 31), medium (E 3), or large (E 1) quartz spectrographs of Adam Hilger, Ltd., a slit width of 0·03 mm. is used. This is the slit width recommended for many years in the instructions for use of the Hilger sector photometers.

In this design certain pitfalls are avoided which lie in wait for the user of spectrophotometers.

(1) In any form of photometer which in its working diverts two beams of light from the single source in order to get them to pass through two different paths (so that an absorbing substance may be placed in one of them, and a comparison substance in the other) it is desirable to ensure that the beams which are utilised originate from the same aspect of the light source. In Fig. 3 is shown how, when the beams from a light source are collected by two separated deviating prisms, as has

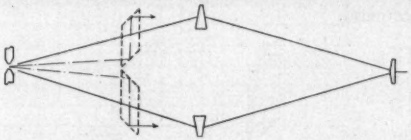


Fig. 3.

been customary in some spectrophotometers, there is a danger that the intensity of illumination from the two aspects may be unequal, since the light source is viewed from different aspects. This in itself is not so very serious, but with fluctuations in position of the light source this difference itself may be variable. In the photometer described the two beams are obtained from one and the same aspect of the light source to a very close approximation, as indicated by the dotted lines in Fig. 3.

(2) In using a rhomb system for diverting and bringing together beams of light, it is usually the case that the faces of the rhomb produce either a shadow or a reflection. This is illustrated in Fig. 4. In the new photometer the angles of the rhomb are so selected that ample clearance is provided between the wall surfaces of the rhomb and the transmitted beam.

(3) Condition (1) being attained, it is safe to assume that the flux of energy from any part of the light source is uniform over the areas of both the apertures *E*, *E*. If the lenses *G*, *G* and the rhombs are large enough all the energy which passes through these apertures is concentrated on the top or bottom part of the slit. If, further, condition (2) is observed, a small element of the slit just above the dividing line and a like small element just below it are illuminated with intensities proportional to the areas of the apertures.

As we have seen, the beams through the slit pass on, without any vignetting in

the interior of the spectrograph, to form images of the apertures at or near the prism, thence to be concentrated in the monochromatic images of the slit at the photographic plate. We may therefore be sure that parts of each spectrum line just above and just below the dividing line have intensities proportional to the areas of their respective apertures E, E.

It is obvious that the more stigmatic the spectrograph (or, to be precise, the more uniform the astigmatism throughout the spectrum) the more closely will the ideal be attained of a sharp division of the two halves of the spectrum line immediately above and below which the intensities of radiation falling on the photographic plate are proportional to the areas of the apertures.

(4) In the examination of many substances, particularly those of interest in biochemistry*, speed of working is important. In the rotating sector type of photometer which has been so much used for this work, the fullest exposure that can be given is only maintained during half the revolution of the sector. Should a polarising photometer be used there is a similar loss of light, since the polarisation occasions a loss of half the radiation. In the photometer now described the full aperture can be used, and for this cause it is almost exactly twice as quick as in the other forms mentioned.

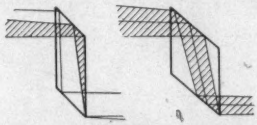
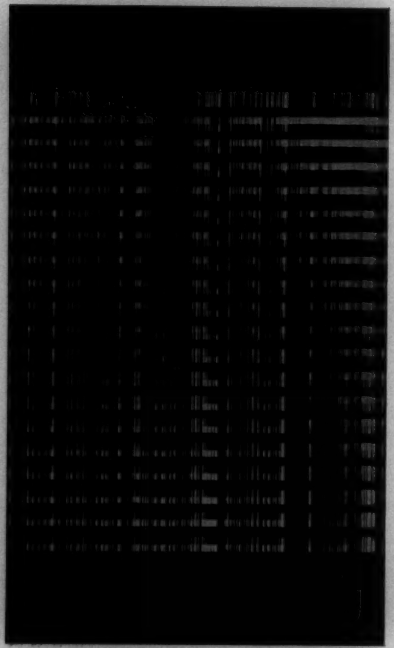


Fig. 4.

(5) Although investigations of the rotating sector form of photometer have repeatedly affirmed its accuracy within the conditions of experiment (see, for instance, references given in Twyman and Simeon, *Trans. Opt. Soc.* **31**, 1930 (169)), it is an undoubted objection to this form of instrument that there is a possibility of fluctuations in the light sources to a greater or less extent synchronising with the rotation of the sector, so that the radiation permitted to pass the sector is not merely a function of the aperture. This, of course, is entirely avoided by the new photometer, in which the exposure is continuous. In the Judd Lewis sector photometer appreciation of this point was shown.

There is an alternative instrument in which, instead of adjusting the relative size of the apertures E, E by the screw and drum D, circular apertures of various sizes are supplied which can be rapidly placed in position to limit one of the beams in a known way. Twenty such apertures are provided, each having an effective density compared with the one preceding it of 0·075: in other words the radiations passing through successive diaphragms in this series have the ratio 1 : 1·188. In one form of the instrument these densities are arranged in a circular disc, so that any one of them can be rapidly brought into position.

* In which oxidation, or other changes due to lapse of time or outside influences (such as irradiation) may occur.



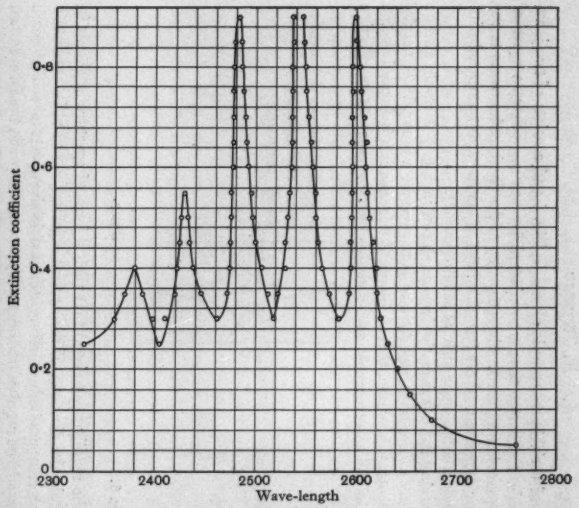


Fig. 5. Absorption curve of benzene (in hexane). Measurements made on "Spekker" photometer used in conjunction with a Hilger E 316 quartz spectograph: strength of solution 1 in 3300; length of tube 2·0 cm.; exposure 5 sec. minimum; Wellington Anti-screen plate. The photograph is a × 2 enlargement of the original.
The photograph and curve shown were obtained with the "Spekker" photometer used in conjunction with a Hilger E 316 quartz spectrograph. Some of the points do not lie on the curve, but it is believed that the accuracy of the instrument is such that were more points on a curve obtained by the taking of an additional plate, it would be found that the points referred to are correct and that richer detail in the curve would thereby be revealed.

Another form of variable aperture is shown in Fig. 6 and consists of two jaws, each having two edges at 90° so that as one jaw slides over the other an aperture in the form of a square is produced. Since the area of the aperture increases as the square of the linear movement of the jaw, this form of aperture, as compared with a plain slit, produces a more open scale at small openings.

By suitably curving the edges of the jaws as shown in Fig. 7 it would also be possible to increase the area of the aperture so as to produce a linear density scale. It must not be forgotten, however, that an image of this aperture is formed on the camera objective of the spectrograph, and that this image represents its effective

Fig. 6.

Fig. 7.

aperture. It will be readily appreciated that such a form of aperture as shown in Fig. 7 formed within the diameter of the camera objective would at its maximum opening represent a comparatively small area of the objective aperture, so that much longer exposures would be required as compared, for instance, with the form shown in Fig. 6 in which the aperture is more efficiently used.

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DISCUSSION

Mr J. G. Holmes: For some time recently I have had the privilege of using a Hilger sector photometer in connection with measurements of the ultra-violet transmission of glass and I have experienced considerable trouble due to flicker of the source. I found, however, that the mean error of a series of observations could be reduced from about 5 per cent. to between 1 and 2 per cent. by replacing the

source by a greyed quartz plate and a suitable diaphragm on which an image of the source is focussed by a quartz lens. I should like to ask the authors if there is any corresponding advantage in using this method with the "Spekker" photometer.

Another question which occurs to me is whether any adjustment of the distance between the apertures *E*, *E* and the lenses *G*, *G*, as in Fig. 1, is required when the two absorption tubes *F*, *F* are removed and a glass specimen is placed in the appropriate beam. I frequently have to obtain transmission curves of specimens of widely differing thickness and, as it would not be practicable to keep a range of quartz plates to equalise the optical paths in the two beams, I wonder whether this asymmetry of the system would decrease the accuracy of the photometer and whether it is possible to correct the balance by alteration of the focus of the condensing lenses *G*, *G*.

Mr J. Guild: It would be of great assistance if a statement were included in the paper regarding the accuracy with which it has been found that the intensity is proportional to the area of the adjustable aperture. I have always regarded this method of intensity variation as only applicable when the source is sufficiently large and uniform that its angular subtense at the lens more than compensates for the chromatic and spherical aberrations of the latter. In the present instrument the source is necessarily small and, with uncorrected quartz lenses, the image on the slit must be considerably out of focus for some parts of the spectrum. In such a case the different zones of the aperture must contribute very unequally to the radiation entering the slit of the spectrograph.

Mr L. Bellingham (communicated): An instrument very similar in design to the "Spekker" photometer was made by Bellingham and Stanley, and fully described by Mr F. R. Lankshear in the *Mem. and Proc. of the Manchester Lit. and Phil. Soc.* **60**, No. 10, 1916. This instrument, instead of having an adjustable slit in front of the first condensing lens, had a rotating disc with an adjustable sector extending to the centre of rotation. It was found in practice that the formation of the spark image on the slit was not satisfactory, as the fundamental conditions for even illumination of the slit was not fulfilled, and is quite unsuitable for use with an arc. Further it is not correct to assume that a condensing lens is evenly illuminated by a flickering light source. Also, owing to the curved surface of the lens, the centre must transmit a greater amount of light than the edge. For these reasons the instrument of 1916 was fitted with the aperture described above, by which all portions of the lens are used to illuminate the slit.

The point raised of the spark being viewed from different aspects if deflecting prisms are used does not in itself give trouble but the dispersion introduced by the use of deflecting prisms does give rise to difficulties in adjusting the instrument. This has been fully dealt with in a paper by H. J. McNicholas in the *Bur. Stand. Journ. of Research*, **1**, No. 6 (942–9).

Mr Twyman: In reply to Mr Holmes the conditions of illumination with the "Spekker" photometer are essentially different from those when using the Hilger sector photometer. With the latter, images of the light source are formed on the

prism of the spectrograph by the two beams into which the light is divided, each of these images consisting of a small spectrum the one with the violet uppermost, and the other with the red uppermost. Thus exact coincidence of the two images is impossible, and if in the fluctuation of the light source the images wander near the margin of the aperture of the prism the violet or the red end of the one spectrum may we weakened relative to the other by the conditions of illumination. With the "Spekker" photometer, images of the fixed and movable apertures are formed at the prism aperture, and these do not vary in position with variations in the position of the light source. It is true that with excessive variations the image of the light source may wander from the slit, with consequent weakening of *both* beams of light; but since the contiguous parts of the two spectra derive their light from the same part of the light source, and since the parts of the images of the latter on the slit where the division by the quartz rhomb takes place are from closely adjacent points of the light source, therefore the wandering of the light source introduces no relative variation in the two spectra near the division between the two.

With regard to the effect of using in the space between E and G (Fig. 1) tubes, or specimens of glass, etc. of widely differing thickness, I am glad Mr Holmes mentioned this point, as it is one which may readily escape the attention of anyone fresh to this work; and as Mr Holmes points out it is not practicable to provide a range of quartz plates to equalise the optical paths of the two beams.

If the light source A is at such a distance from L that the light from any point of A is strictly collimated, then absorbing substances of differing thickness may be compared without any compensation. But since, owing to the expense and rarity of optical materials (such as fluorspar) suitable for achromatising quartz, the lenses L are not customarily achromatised, measurements should, therefore, in such cases be repeated for the various wave-lengths, with the light source focussed for each wave-length. Similarly, the lenses G, G should also be re-focussed for each wave-length, so that a well defined image of the light source may be formed for that wave-length on the slit of the spectrograph.

There is also a variation in the position of the image of the apertures E, E due to the chromatic aberration of the lenses G, G and the maker's attention will be directed to seeing that the user is warned of the limits to which this can safely be permitted.

Mr Guild's remarks are very pertinent. As he will see, the direction of my thoughts has been contrary to his in that I have assumed that the user would remove the effect of chromatic aberration by re-focussing the lenses. It may be that by following Mr Guild's line of thought (a means for which is to be found in the device of a greyed quartz plate referred to by Mr Holmes) the user could be saved this trouble, and I shall investigate the matter.

I am interested to hear from Mr Bellingham of the variable aperture photometer made by Messrs Bellingham and Stanley in 1916. I was not aware of this.

With regard to Mr Guild's question regarding the accuracy with which the intensity is proportional to the area of the adjustable aperture, all I can say at the moment is that it has not been possible to detect any certain differences between

measurements of absorption made with this and with the Hilger sector photometer. It is not intended to leave the matter in this position, which cannot be considered ideally satisfactory, and experiments are now in progress in the first place to apply microphotometric methods to obtain a more sensitive comparison of the intensities, and then to endeavour to make measurements with the new photometer to the higher accuracy thus obtainable. I refrain from mentioning the ways I have in mind for checking independently the measurements so made, but hope to have the benefit of Mr Guild's criticism thereon on a future occasion.

A NEW PHOTO-ELECTRIC SPECTROPHOTOMETER

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Freedom Research Fellow

and ENSOR R. HOLIDAY, B.M., M.R.C.S.,
Grocers Scholar

(Research Laboratories, The London Hospital)

[Communicated by Mr F. Twyman.]

MS. received, 24*th July*, 1931. *Read and discussed*, 10*th December*, 1931.

ABSTRACT. The photometer described makes use of a device by means of which absorption measurements may be made in the ultra-violet region of the spectrum without recourse to calibrated wedges or sectors. The hydrogen discharge tube is used as the source of radiation. A photo-electric bridge is employed, one arm of which is controlled by a small fraction of the light falling on a third cell; the current from the latter may be measured galvanometrically. The two fractions are kept in constant proportion over the range of intensities used.

Introduction

The photometer here described was developed with three objects in view:

I. To be sensitive enough for use with the hydrogen discharge tube.

II. To avoid the use of mechanical sectors or wedges.

III. To provide a rapid method for accurately measuring absorption at a given wave-length.

I. The hydrogen discharge tube has the advantage over other sources of ultra-violet light in that it provides a continuous spectrum between 6000 and 1800 Å., so that absorption measurements may be made, at any wave-length, between these limits. The energy of its emission is low as compared with the mercury vapour lamp, but the advantage of the continuous as against the intermittent spectrum for absorption measurement is evident. Several photo-electric methods of measuring light intensities of the order of those from the hydrogen discharge tube are available but two of these are preferable: (*a*) the compensation by induction method, (*b*) the balanced Wheatstone bridge method. The former method, although at first sight simpler, has in the writers' experience several disadvantages which decided them in favour of the latter:

(i) The difficulty of accurately repeating time of exposure.

(ii) The length of time between the exposure through water and that through the first solution, rendering the liability to error through variation of the light source great.

(iii) The after-discharge experienced with a gas-filled cell prevents the balance point being sharp, as there may be a slow variable drift of the electrometer after the cell has ceased to be exposed.

(iv) The smaller latitude in dealing with dissimilar energy values of different parts of the spectrum.

II. Whether the mode of varying the compensating light be by means of sectors or of wedges the same difficulty is experienced with both, that is, of accurately calibrating them. Another disadvantage which applies particularly to the sectors is the more or less complicated optical train required for manipulating the light beam to be varied. With a source of radiation like the hydrogen tube, one cannot afford to lose much light by reflections at lens surfaces. As for each surface the loss in this way is anything from 4 per cent., it can be seen that the optical train of an instrument such as a Judd Lewis sector will reduce the light from the source very considerably.

The method aims at measuring the compensating light electrically, measurement depending on the reading of a potentiometer. Thus all mechanical means of measuring the light are avoided.

III. Using the method described an estimation requires about one minute and the error is less than 1 per cent.

Description of Apparatus

The central unit of the apparatus consists of an airtight metal box dried with P_2O_5 and painted black inside. It contains three photo-electric cells *A*, *B*, *C* arranged in the manner shown in Fig. 1. In the box is also placed a Lindemann electrometer *E* and an earthing switch. The photo-electric cells are light-shielded from one another. The principle employed in measuring the weak compensating light falling on the cell *B* is to make this beam a constant fraction of a beam strong enough to be measured with a photo-electric cell in series with a galvanometer. This is obtained by placing the plane glass *M* at an angle of 45° in the beam from a 36-watt car lamp *L*, so that a fraction is reflected on to the kathode of the compensating cell *B*. The main beam passes on to strike the kathode of a potassium cell *C*, connected in series with a sensitive moving-coil galvanometer *G*.* Between the car lamp and the mirror *M* is a neutral grey wedge W_1 for varying the intensity of the beam. Across the terminals of the galvanometer is also connected a potentiometer *P* acting in opposition to the photo-electric cell *C*.

The light from the hydrogen discharge tube is focussed by a cylindrical lens on the slit of a Hilger quartz monochromator. The emergent beam falls on the kathode of *A*.

* Tinsley moving-coil galvanometer, resistance 4000 ohms.

Procedure

Water is placed in one of two quartz cells, and the solution, whose absorption is to be measured, in the other. These cells are held in a slide so that either one or the other can be brought into the monochromatic train.

The galvanometer is switched out of circuit and the scale adjusted accurately to its zero mark. The wedge W_1 is set at a position of maximum transparency, and by means of the second wedge W_2 the electrometer is brought to its balance point. The galvanometer is now switched in and adjusted to the zero mark by means of a resistance R_3 in series with the potentiometer, which is set at 100. The potentiometer reading is therefore a measure of the emission from the cell C, which in its

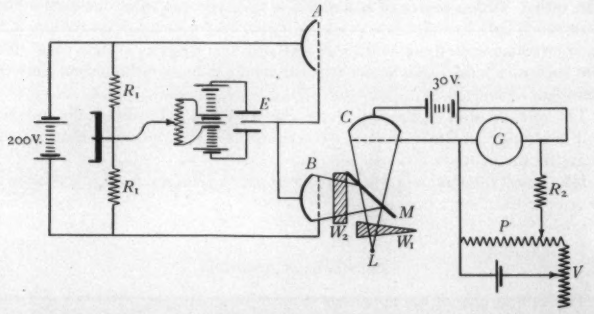

Fig. 1. Diagram of connections. A, sodium gas-filled cell; B, C, potassium vacuum cells; E, electrometer; G, galvanometer; L, 12-volt 36-watt car lamp; M, plane glass plate; P, potentiometer; R_1, R_1, 20,000 ohm wire-wound resistance; R_2, 1 megohm resistance; V, variable resistance; W_1, W_2, optical wedges.

turn is proportional to the light falling on B. The cell containing the absorbing medium is now made to replace the water cell. The galvanometer is switched out to avoid the excessive swinging which would alter its zero position and the electrometer balanced again by moving the wedge W_1. The galvanometer is switched in and brought to its zero position by means of the potentiometer. The new reading of the potentiometer gives the relative transmission of the absorbing medium as compared with water. By trial the potentiometer can be set at roughly the new reading before the galvanometer is switched in, thus avoiding swinging.

For ideal conditions it is necessary that both light sources should be constant. This is obtained for the car lamp by charging the accumulator at almost the same rate as it is discharging. In practice the lamp takes about 3 amp. and the accumulator is charged at about 2·5 amp. Under these conditions there is no visible variation (i.e. less than 0·1 per cent.) of the galvanometer over a period of 5 minutes.

Regulation of Hydrogen Discharge Tube

The mains in the writers' laboratory are D.C. and are coupled to a small rotary converter supplying 110 volts A.C. to a transformer stepping up to the 2000 volts required by the discharge tube. The writers have found that a battery of accumulators, placed across the mains supply to the rotary converter, of a voltage that will cause them to discharge at about $\frac{1}{2}$ amp., keeps the emission from the tube constant to within 1 per cent., whereas the variation of the mains is about 10 per cent. Any variation is a gradual one and is very small over the time (15–20 sec.) between two balancings of the electrometer.

Practical Considerations

There are one or two practical points to be observed.

(1) With regard to the splitting of the beam to the photo-electric cells *B* and *C*, it is essential that the whole of the divergent beam falling on *C* should be represented on *B*. This is because the beam passing through the wedge is of finite diameter and all portions of the beam are not of equal intensity.

(2) The wedge W_1 must be neutral. The wedge made for the writers by the Ilford Plate Company seems perfectly satisfactory in this way. The limits of transmission of the wedge used are 100 to 1 per cent. The wedge W_2 should be of the same range.

(3) The earthing switch (Fig. 2) is designed to minimise break charges. A rod of brass *A*, connected to the electrometer needle, is embedded in sulphur and supported by the case *B*, and has a flat tip of gold soldered on. The other end of the case supports a thick brass collar *C* in which slides the brass plunger *D* which is also gold faced. The plunger is permanently earthed and by means of the springs *E*, *E* the two gold faces are kept in contact unless the plunger is depressed by applying tension to the wire *F*. A small hole in the side of the case *B* allows communication between the air of the main box and the space round the contacts which are thus kept dry. The collar *C* is a tight sliding fit in the case and allows the contacts to be cleaned with alcohol and ether before assembling. It is found that, with such a switch, the needle is deflected less than half its width when the plunger is depressed.

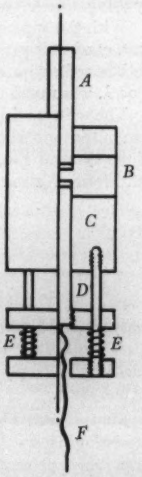

Fig. 2. Earthing switch.

Tests

The following simple tests were applied to the apparatus.

(1) The 36-watt lamp was replaced by two small 6-watt lamps connected in series. The positions of these were adjusted so that the light from each one separately, falling on the cell *C*, required the same opposing potential from the potentiometer to keep the galvanometer image at zero. Exposure of the cell to the light of both

lamps now required exactly double the potentiometer potential to keep the galvanometer at zero. This experiment was repeated with the lamps at a nearer or greater distance from the cell *C* and the proportionality law was found to be followed for all light intensities investigated.

(2) To find whether the Wheatstone bridge circuit also followed the proportionality law, a similar test was made, the addition of two more car lamps being made in front of the window of cell *A*. Each of these lamps was made to match one of the former lamps by adjustment of its distance from the cell. It was found that when all four lamps were then exposed the bridge remained balanced. This showed that the circuit obeyed the proportionality law.

(3) The neutrality of the wedge W_1 was tested by determining the absorption of a solution using widely different light intensities and therefore different parts of the wedge. No difference within the limits of error of the apparatus could be detected by using different parts of the wedge.

With the apparatus described above, transmissions of between 40 and 50 per cent. can be measured with an error of less than 1 per cent.

The writers have found that the sodium cell is sufficiently sensitive from 5000 to 2500 Å. when used with the apparatus.

The writers wish to express their thanks to Messrs Adam Hilger, Ltd., for the loan of the monochromator during the experimental stages of the work, to the staff of the General Electric Company's research laboratories, and to Messrs Tinsley and Messrs Watson for loans of apparatus.

DISCUSSION

Prof. A. O. Rankine: Dr Holiday stated in his description that the amounts of light received by the photo-electric cells *A* and *B* (Fig. 1) are equal. May I ask if this is actually so? The gas-filled cell *A* is presumably chosen because of its greater sensitivity as compared with the vacuum cell *B*, and their characteristics are therefore not the same.

Mr J. Guild: The Optical Society is to be congratulated on having had several interesting papers dealing with spectrophotometric methods for use in the ultra-violet region presented to it in the last few months. The subject is one which merits all the attention various workers are devoting to it. It is always desirable, however, to accept with caution the results given by any new type of equipment, particularly one involving photo-electric cells, and to avoid the mistake of confusing precision with accuracy. At first sight nothing should be simpler than to obtain a true value for the ratio of two intensities of homogeneous radiation by means of photo-electric cells; yet experience in all the standardising laboratories of the world indicates that photo-electric spectrophotometers should only be trusted in so far as the results they give can be shown, by critical tests, to agree with those obtained by other methods. The authors are to be congratulated on having produced a compact

piece of apparatus embodying some very ingenious features. I notice, however, that in the photo-electric bridge a gas-filled sodium cell is balanced against an evacuated potassium cell, and that the measurement assumes the same proportionate diminution of the current from each for the same proportionate reduction in the intensity of illumination. It seems to me impossible that this similarity in the characteristics of two cells of completely dissimilar type can hold over any useful range of intensity, and I would suggest that it is absolutely necessary in a system of this kind that all three cells should be of identical type.

The authors refer to the difficulties involved in the use of sector discs, partly on account of difficulty of calibration and partly on account of loss of intensity in the auxiliary optical apparatus required with them. Presumably they are not here referring to rotating sectors, which, of course, require no auxiliary optical apparatus and, except for comparatively low values of effective transmission, no calibration. The legitimacy of using such sectors with photo-electric cells has been established by various workers.

Mr T. Smith: Will the authors give particulars of the hydrogen discharge tube used? Have they considered the use of a large tube emitting enough light to enable a vacuum cell to replace the gas-filled cell at *A*?

Messrs Smith and Holiday: In reply to Prof. Rankine the amounts of light received by cells *A* and *B* are not equal, but when the bridge is balanced the currents passed by the cells are obviously equal.

The authors are well aware of the shortcomings of the gas-filled cell. As the energy available in their laboratory was not sufficient to energise a larger hydrogen tube, they had to use it owing to its greater sensitivity. With regard to the cells being dissimilar in type, the response of the gas-filled cell was found to be linear over the range of intensities used, and the employment of the vacuum cell may be regarded as a convenient method of varying the opposing arm of the bridge. Empirically the arrangement has been found to be reliable. The method suggested by Dr N. R. Campbell to test this has been described in the paper.

The hydrogen tube used was of German manufacture and obtained from Messrs Adam Hilger, Ltd. It required about 4000 volts to energise it and passes 300 to 400 mA. It was water-cooled. The necessary main supply to energise a larger tube is not available in the authors' laboratory.

METHODS OF FOCUSSING PROJECTORS

DESCRIPTION OF A DEMONSTRATION

By J. M. WALDRAM, B.Sc. (Eng.)

Communicated from the Research Staff of the General Electric Co., Ltd., Wembley, England

Demonstrated, 10th December, 1931.

In many projecting systems used commercially it is necessary, on account of price restrictions and of the difficulty of forming really accurate reflecting surfaces, to use mirrors having departures from the ideal contour which are large compared with those existing in optical systems, although the mirror is still capable of producing a useful beam for the purpose for which it is required. Such mirrors may be formed of spun or pressed metal, or blown, pressed, or bent silvered glass. They have really no very precise focus in the optical sense, and the light source has up to the present been

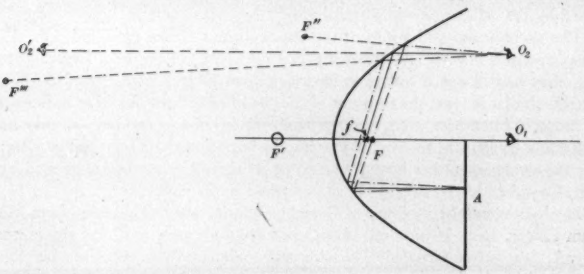

Fig. 1.

set at the optimum position by the method of trial and error by observing the appearance of the beam produced. This method is sufficient for many purposes, but it was found in the case of a large aerodrome floodlight, which consisted of nine trough reflectors of bent silvered glass each having a 1000-watt line filament lamp, that it was a matter of great difficulty to get all the beams to superimpose to the best effect, largely owing to the fortuitous nature of the errors in the mirrors. A method of focussing was called for which would take into account these errors, and would enable the lamps easily to be set at the optimum position in daylight.

After some trials with different methods the following was devised. A vertical

plane mirror A (Fig. 1) is placed at right angles to the axis of the trough, with its upper edge on the axis. This can be set geometrically with sufficient accuracy. The observer places his eye at O_1 and, if the lamp filament is in place, will see the filament F, and also a magnified image of it, F', formed in the trough mirror. If the filament is now raised or lowered, a position can be found when the filament, the image of the filament, and the top of the auxiliary mirror A appear to be in line. The filament is now in the plane of the axis, but not necessarily at the focus.

The eye is now raised to a position such as O_2. In such positions an image of the observer's face will be seen, formed as shown in the figure by three reflections in the trough and one in the plane mirror. The rays forming the image O_2' of the eye will be seen to pass twice through the focus f, if the trough is accurately formed, and the line joining the eye to its image will be seen to be always parallel to the axis of the trough whatever the position of the eye.

If the filament is in position, and if it is not at the focus, two images of it, F'' and F''', will be seen which always lie one above and one below that of the eye; and if it is moved to and fro a position can be found in which the two images and that of the eye coincide. The filament is then at the focus. If the filament is not parallel with the length of the trough, the images seen will not be mutually parallel. In the case of a large reflector it is possible to see an image of the horizon as well, so that the whole trough may be orientated, if desired, with respect to it.

It will be seen that F'' is formed after one and F''' after three reflections in the trough. Consequently, departures from the ideal parabolic contour show themselves as irregularities in the images formed; generally it is not possible to make the images coincide over the whole of the aperture of the mirror as the head is moved up and down. In that case it is possible to set the filament so that the most part of the mirror is in focus, or to reject the mirror as unsuitable.

A paraboloidal mirror such as a headlight mirror can be very easily focussed in daylight in this way, using a piece of plane mirror held against the front glass. In the case of a surface of revolution, as compared with a trough, the first adjustment of placing the source on the axis can be performed at the same time as the second. An image of the eye is always seen with two images of the filament; if the latter is not on the axis the two lie one on either side of the eye and not in line, and if it is out of focus, the two are in line but not coincident. The imperfections of a headlight mirror make the second image rather difficult to see, but it is sufficient to regard the image of the pupil as being at the focus and to set the filament on it. Thus either end or any desired part of the filament can easily be set on the focus.

It will be seen that this simple method can be applied only to such parts of the mirror as lie beyond the latus rectum. In the case of shallow mirrors it will not, of course, work, but it is possible to use a spherical mirror to the same effect. In Fig. 2 a spherical mirror is placed with its centre of curvature at the focus of the main mirror. It is then a simple matter to set a source at the focus, for an inverted image will be seen which can be brought accurately into coincidence with the source itself, the two being observed by reflection in the main mirror. The two mirrors may be set relatively to one another by using a strip of plane mirror A placed at right angles to

the axis of the main mirror. If the spherical mirror is placed with its centre at the focus of the main mirror, an image O' of the eye will be formed as before, the line joining it to the eye being parallel to the axis. This image and that formed by the strip of mirror, O'', will then appear to be in line, and will remain so when the eye is moved up and down. If the spherical mirror is not correctly placed, a displacement will occur, the former image being either above or below the latter if the centre

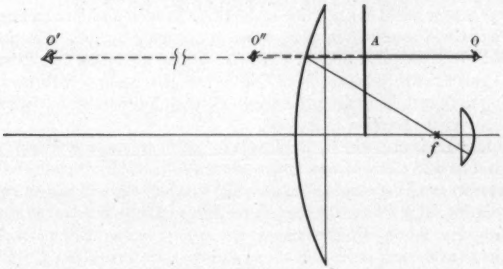

Fig. 2.

of curvature is on the axis but not at the focus, or closer to it in some positions of the eye than in others, if it is off the axis. This adjustment could be made once for all, and provision made for the spherical mirror to be removed and replaced in some locating device, or else left in position as a backing mirror, when it will cut off spill light from the bare source and increase the beam intensity.

EXHIBIT OF MICROSCOPES

By Messrs W. WATSON & Sons, Ltd.

Exhibited, 10th December, 1931.

The following instruments, which show some novel features of design, were exhibited:

The "Service" microscope.
The "H" Edinburgh student's microscope.
The works model of "Service" microscope.
A low-power stereoscopic dissecting microscope.
A modification of the usual form of eye-piece camera.

"Service" and "H" Edinburgh Student's Microscopes

In these instruments (Figs. 1 and 2) the optical bench type of limb, which was first introduced by us in 1919, has been carried to its logical conclusion. The dovetailed surfaces for the fine adjustment motion and for the substage are now machined in one operation. The limb therefore forms an accurate optical bench, and absolute alignment is assured. At the same time the limb has been substantially modified and is now considerably thicker than hitherto.

The spiral focussing screw for the condenser has been discontinued, its place being taken by a rackwork substage which can be obtained in three forms: (*a*) with a simple tube for the condenser, (*b*) with centering screws and swing-out to the optical part of the condenser, (*c*) with condenser changers and centering screws, thus giving facilities for centering both the iris diaphragm and the optical part of the condenser relative to the optical axis of the instrument. The substage focussing adjustment can be made from either side of the limb.

The design of the body tube represents an outstanding advance. Hitherto the rib which forms the bearing for the coarse adjustment has been screwed or soldered to the body tube. In the instruments exhibited, the body and rib are cast in one unit, the body is then drilled out, and a tube is thus formed. The dovetailed slides of the coarse adjustment are machined to the axis of this tube, thus ensuring parallelism.

The usual form of draw-tube has been discontinued and a large draw-tube, 36 mm. in diameter, is now employed. The tube is provided at one end with a fitting to take the standard size of student's eye-piece. The eye-piece is suspended in the tube, and therefore light scattered by internal reflections in the tube is prevented from entering the eye-piece. At the other end of the tube, and in one piece with it, is a fitting having the standard R.M.S. thread to carry microscope objectives, photomicrographic objectives, analyser prisms, etc. The draw-tube is doubly engraved to show the tube length with and without the nose-piece.

Works Model of "Service" Microscope

In many laboratories of small works a microscope is required which can be used both with reflected light, as in metallurgy, and with transmitted light. The instrument exhibited has been designed with this end in view. It has the conventional stage

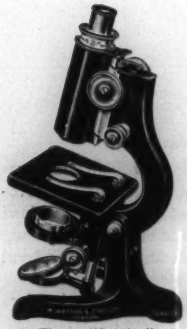

Fig. 1. "Service" microscope.

Fig. 2. "H" Edinburgh student's microscope.

which is, however, fitted with rackwork focussing and a central removable disc. Below the stage is mounted a centering substage having rackwork focussing and an independent mirror box with plane and concave mirrors. The maximum gap between the surface of the stage and the nose-piece exceeds 6 inches.

Low-power Stereoscopic Binocular Dissecting Microscope

The demand for an instrument of this type for laboratory demonstrations and University purposes increases every year. The model exhibited shows some novel features. Instead of two separate prism boxes the prisms are mounted in one box with rackwork adjustment for interpupillary distance, and the necessity for cemented

prisms has been overcome. Difference in focus of the two eyes can be compensated by an adjustment fitted to the eye-piece tube. Double slides are provided, one rack-work and one plain, so that the body with eye-pieces and objectives can be removed and mounted on any other suitable fitting. The standard mounting is on a rigid stand with horse-shoe foot, glass stage, and telescopic pillar.

The whole apparatus can be mounted above a dish or other specimen, and planes 1 inch below the level of the table can be focussed, while objects up to $2\frac{1}{2}$ inches in thickness can be mounted on the stage and examined with ease. The magnifications provided are 12, 18, 36, and 48, obtained by means of two pairs of objectives and two pairs of eye-pieces. The objectives are mounted in a nose-piece.

The transmission of light in this model is particularly good, and the field is considerably larger than in the conventional Greenough pattern. The instrument is rigid and compact, and it may be used by those unaccustomed to handling models of this type without risk of its adjustment being disturbed. The mounting of the objective tubes is a complete departure from convention, the usual three bearing screws being replaced by another device.

Eye-piece Camera

This model is designed for use with quarter-plates. The camera not only fits into the microscope tube but is also clamped externally at three points. The plate holder, instead of being fitted to the centre of the shutter as is customary, is carried on the microscope itself, as the mounting rises direct from the clamp above the microscope eye-piece. In this model the shutter is let into the camera, thus obviating the risk of vibration.

REVIEWS

The Mathematical Papers of Sir William Rowan Hamilton. Vol. I. *Geometrical Optics*. Edited for the Royal Irish Academy by A. W. CONWAY, D.Sc., F.R.S., M.R.I.A., and J. L. SYNGE, D.Sc., M.R.I.A. Pp. xxviii + 534; frontispiece. (Cambridge: University Press, 1931.) Price 50*s*. net.

The first half of the nineteenth century was remarkable for the number of men who contributed to the progress of optical science. Before the days of Helmholtz, Abbe, Clerk Maxwell, and Lord Rayleigh, to name only a few of the leaders of the nearer past, we find among continental workers such men as Fresnel, Fraunhofer, Arago, Porro, Cauchy, Petzval, Gauss, and Daguerre, and at home an equally splendid list, Nicol, Brewster, Wheatstone, Lister, Herschel, Airy, Coddington, Young, and Hamilton. Of all these illustrious men there is, perhaps, none whose optical accomplishments were quite as magnificent as Hamilton's: yet it is probably also true that there is no other whose work is so little known at first hand.

The general neglect of Hamilton's work will be appreciated if we contrast the attention given to Hamilton and to Young in modern optical treatises. Both men were endowed with gifts which enabled them to excel in diverse ways. Young's contributions to optics were much slighter than Hamilton's, but they are mentioned in almost every textbook. Hamilton's work, probably the most fundamental that has ever been carried out in optical theory, is rarely mentioned and never adequately presented.

It is difficult to suppose that Hamilton's optical papers were neglected because they were overshadowed by his work in other subjects. It is much more likely that the neglect is due to their nature, or to current opinions on their nature, and to their supposed inaccessibility. The last factor, at least, is no longer of importance, for the optical writings are brought together in this first volume of the collected papers. The frequency with which references to Hamilton's papers occur at the present day in publications on mathematics and mathematical physics leaves little excuse for the not uncommon assumption that he dealt only with matters too abstruse to be of interest to any but the most academic of physicists.

Although Hamilton is known best for his dynamical work and as the originator of quaternions, his early papers were all devoted to optics. The dynamical developments, in fact, grew out of the earlier optical investigations. Like all the other papers he published, these optical papers deal only with problems of the most general nature. He desired to establish methods powerful enough to deal with whole groups of problems, not to produce solutions of specific problems. We learn something of his aim from his own words (p. 315):

> Those who have meditated on the beauty and utility, in theoretical mechanics, of the general method of Lagrange—who have felt the power and dignity of that central dynamical theorem which he deduced, in the *Mécanique Analytique*, from a combination of the principle of virtual velocities with the principle of D'Alembert—and who have appreciated the simplicity and harmony which he introduced into the research of the planetary perturbations, by the idea of the variation of parameters, and the differentials of the disturbing function, must feel that mathematical optics can only *then* attain a coordinate rank with mathematical mechanics, or with dynamical astronomy, in beauty, power, and harmony, when it shall possess an appropriate method, and become the unfolding of a central idea.

It is this idea, this method, that Hamilton has presented to us in his paper on "Systems of Rays" and its three supplements. Of the result the editors of this volume write "Hamilton was able to present the whole theory of geometrical optics in a form so compact that its application appeared almost impossible," and again "The application of Hamilton's method to the optical instrument of revolution was for long the despair of mathematicians."

Hamilton's apparent interest only in the development of a general theory has not escaped adverse criticism. For instance Lord Rayleigh, who showed by examples how readily the method could be applied to important physical problems, said

> Hamilton allowed his love of generality and of analytical developments to run away with him. In his Memoir on Systems of Rays, with its elaborate and rambling supplements, there is little to interest the practical optician, though the mark of genius is throughout apparent. It was only in two or three pages of a later paper that he applied his powerful methods to the real problem of optics. As Finsterwalder has remarked, his "six radical constants of aberration," expressing the general properties of a symmetrical instrument, are at once an anticipation and a generalisation of von Seidel's theorems. But the published work is the barest possible summary. If Hamilton had been endowed with any instinct for optics proper, he could have developed these results into a treatise of first-class importance.

Until the appearance of this volume there was every reason for regarding this as a fair criticism of Hamilton's optical work. True, friends had recorded that he showed remarkable zeal in carrying out numerous calculations to test his theories, but there was nothing to show that this related to his optical investigations. We now know that the charge is mistaken, but Hamilton himself is responsible for our misjudgment through withholding from publication descriptions of any of the applications he carried out. Fortunately the voluminous manuscripts he left have been carefully preserved at Trinity College, Dublin, and from these the editors have selected extracts for publication. Seven of these appear in this volume. Those of the greatest interest are his first paper "On Caustics," a short extract from a notebook (probably written in 1833) dealing with the aberrations of an optical instrument of revolution, two letters to Professor John Phillips on the construction of object glasses, and an extensive investigation on the improvement of the double achromatic object glass.

Hamilton's first paper is concerned with a geometrical, rather than a specifically optical, problem. For most readers it will be of interest mainly as the production of a boy of 17 years of age—we are told it was presented to the President of the Royal Irish Academy a year before Hamilton, then 18, entered Trinity College. His next paper—the "Theory of Systems of Rays"—was read when Hamilton was 19 years old. Quite apart from the almost incredible fact that anyone so young should have composed this work, the paper bears clearly the marks of Hamilton's genius. The editors state that the manuscripts they have examined contain nothing which shows how the idea of the Characteristic Function originated. With the new light shed by these manuscripts on his methods of work we may hazard a guess that the course Hamilton followed was not very different from that set out in the opening sections of the "Systems of Rays." With Fermat's theorem generally established, he would naturally construct expressions for the "action" (or path length as we should now say) in special cases; the principal property of the characteristic function would then soon be observed, and the continuation as it is presented in the paper follows almost directly.

The Hamilton we picture with the aid of the papers now first published differs from the man we should have pictured from those previously known. He is far more human, affected by infirmities like our own, and at times he fails to reach the goal towards which he is striving. His achievements are the outcome of unceasing toil. The hardship he has endured and the difficulties he has overcome are not to be inflicted on the world. Only when he has found something of great beauty in which he and we can rejoice alike does he speak to us, and in contemplating his treasure his labours seem to have been forgotten. But

now we have been given a peep behind the scenes. So far from being indifferent to the needs of the practical optician, and having no instinct for optics proper, Hamilton seems to have been in advance of all his contemporaries in these respects. He not only analyses all the usual aberrations accurately, but, in opposition to Herschel, puts forward the principles for correcting object glasses which are followed to-day, and finds the curvatures to be used with given glasses. We can best indicate the interest of his work to a modern reader by making a few observations on a single topic. For this we choose what we now know as coma: Hamilton's teaching on this subject is particularly interesting because the existence of such a defect does not appear to have been previously suspected.

The first relevant reference is in §63 of Part I of the "Systems of Rays," where the general characteristics of this aberration are correctly described. We may therefore assign 1824 as the year in which coma was born. It was named by Lister in 1830 in his well-known paper on microscopic objectives. In notes assigned to the year 1833 and reproduced in this volume, we have a detailed description of first order coma (several diagrams drawn by Hamilton are also reproduced) including the out-of-focus effects and the condition for its removal. The subject naturally arose again when Phillips consulted Hamilton on the best design for some objectives the former wished to have made. This correspondence may have been the cause which led Hamilton to record at greater length the conclusions he had reached on object glasses; of this MS., which was written in 1844, an extract running to 74 printed pages is included in this volume. The advantages of Hamilton's construction over Herschel's are fairly stated, and it is of interest, as showing that Hamilton had studied Herschel's treatise carefully, to read "the theory of my object glass, therefore (at least so far as indiametral rays are concerned), and the fundamental equations which construct it, might have been deduced, although less elegantly, without the introduction of my characteristic function T," the context clearly showing that Herschel's method might have been used. How consonant Hamilton's ideas are with those now held may be inferred from several passages. For example on p. 395 we read: "And $\frac{1}{(\mu - 1) R}$ may not improperly be called the *focal length*, or $(\mu - 1) R$ the *power*, of the lens. This focal length $\times$ the sine of the semi-diameter of a planet, will give the radius of its image formed by the lens." The use of sine rather than tangent is very significant of Hamilton's instinct for taking the right turning. We may remember that Abbe made the other choice. As a final example we note that his name for spherical aberration is "direct aplanaticity" and for coma "oblique aplanaticity." Thus Hamilton used aplanaticity both in its original sense and in the modern sense introduced by Abbe. He, however, avoided the difficulties occasioned by the double usage. We could do far worse than follow Hamilton here.

We have attempted to indicate that this volume of Hamilton's contains much of the greatest interest to opticians. The new matter is only a sample of what exists. To a large extent the contents of the manuscripts are still unknown. In a letter written in 1841 Hamilton, speaking of the work he carried out in 1831, says, "But I am absolutely startled by the amount of calculation, respecting optics, which I committed to paper in the same year.... I have easily resolved, to-day, problems which then required a vast expenditure of labour...." We can only regret that Hamilton was not induced to give the world some account of these not unfruitful labours. That they should have lain hidden for a century is tragic. He knew facts of great importance, some of which were only revealed in 1856 on the appearance of von Seidel's work.

It goes without saying that the papers in this volume are of special interest to others than opticians. They are not easy reading, but they will repay perusal by those with the mathematical knowledge required to appreciate them. The editors have written an introduction which will be found most useful as a general guide, and have added an appendix of 27 valuable notes, which themselves extend over 56 pages. They have done their work thoroughly, and have provided an index which seems to give all the references that will

be required. That the editorial contributions are themselves of wide interest will be clear if we quote the opening statement of Note 20, entitled "On group-velocity and wave-mechanics." It runs

> The relationships between the emission and undulatory theories and the equivalent results yielded by the principles of Least Action and of Fermat, as developed in the Third Supplement, have inspired directly the foundation of Wave Mechanics as treated by de Broglie and Schrödinger. In particular in a paper by the latter the actual genesis of this development of Quantum Mechanics from the optical ideas of Hamilton is given.

and the connection is then discussed in mathematical form.

It is hardly necessary to say more of the volume itself than that it is worthy of the Cambridge University Press.

T. S.

Dynamic Retinoscopy. By MARGARET DOBSON, M.D. Pp. 56; 12 Figs. (Oxford: University Press. London: Humphrey Milford, 1931.) Price *7s. 6d.* net.

Dynamic Retinoscopy bids fair to rival in popularity the subject known under the general title of Ocular Muscles, but also variously called Ocular Calisthenics, Orthoptic Training, etc. It was introduced by A. J. Cross in 1902 as a means of measuring latent hyperopia. As in the case of the ocular muscles, attempts are being made by its protagonists to erect imposing technical structures upon foundations that are by no means well and truly laid.

In a book such as the one under review, devoted to retinoscopy, we might rightly expect to find some theoretical consideration of the principles upon which the extremely useful practice of retinoscopy is based, but the only references to such are to be found in the following lines which appear to be somewhat involved statements without theoretical basis or significance: "The underlying principle is the understanding of conjugate foci, always realising that a point on the macula lutea is one focus, and the artificially made myopic far point, the second conjugate focus, at whatever distance may be chosen by the operator," and "When the mirror is tilted the source of light moves in the opposite direction with a plane mirror, but the fundus illumination moves in the same direction whatever the refractive condition of the eye. Should the reversal point or far point of the myopic eye be in front of the refractionist's eye, the movement appears to be in an opposite direction, because it has come to a focus and crossed before reaching his eye."

It is surprising how few of those who are proficient in the accurate measurement of ocular refraction by means of retinoscopy have any clear understanding of the optical principles of the method, and who in consequence devote their attention to those aspects of the subject that are likely to provide information upon which real progress can be made in attempts to extend the scope of the method. The book before us provides little or no help in this direction. It is apparently intended as a guide to the practice of the art.

In a number of places the author has taken insufficient care in her statements regarding optical matters. We are told that the distance of 1 metre is the standard distance usually employed in static retinoscopy. "Ten centimetres farther from, or nearer to, the patient's eye than 1 metre only results in an error of 0·12 dioptre." The statement that "A prism refracts an object towards its base and displaces the image towards its apex" is surely a novel view of the effects of refraction.

It is usually claimed that the dynamic method of retinoscopy, being employed as a binocular test while the subject's accommodation and convergence are in use, provides an objective method of measuring the amplitude of accommodation as well as the relation between that function and convergence. For this reason a considerable part of the book

has been devoted to the subjects of muscle anomalies and accommodation and convergence. Heterophoria is dealt with at some length and the chapters on this subject contain some useful information, but we feel that one third of the book is an unduly large proportion to have given to it, especially as the relation of much of the work to retinoscopy is not apparent.

The large and well-reproduced photographs of retinoscopes and muscle-testing apparatus are good, but probably less instructive than diagrams of their optical systems and mechanical construction and descriptions of their mode of use.

In considering the amplitude of accommodation that exists while any fixed amount of convergence is in use, the author states that these functions have a definite relation: "The total amount of relative accommodation is equal to 40 per cent. of the amplitude still unused, while the negative relative accommodation is 40 per cent. of that in use." Opinions vary considerably on this subject of relative amplitudes. If we may judge by the increase in relative positive and negative amplitudes which may be acquired by practice, we should conclude that the relation is somewhat less definite than that stated here. This question is important in the interpretation of the results of a dynamic retinoscopy test; it is referred to again in the last chapter which deals with the application of the dynamic method to the determination of the comfortable correction of the patient. The difficulty of the interpretation of the results appears to have been fully appreciated by the author, but the attempt to divide the wide "neutral zone" which is found by this method into latent hypermetropia and relative negative accommodation can only succeed if the latter value is accurately known. We doubt whether the allowance of 1·35 D. recommended here is a sufficiently constant value for the relative negative accommodation at the distance of 13 in. for it to be assumed in all normal cases without further test. Unless this value is accurately known for the particular case under examination, the results of the dynamic test appear to be useless.

E. F. F.

Penrose's Annual: The Process Year Book and Review of the Graphic Arts, Vol. XXXIV. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S. Pp. xv + 126. (London: Percy Lund, Humphries & Co., Ltd., 1932.) Price 8*s*. net.

The present volume contains the following articles: The Editor's Review, Fifty Years of Half-tone, Lettering and the Printer, The Modern-face Type in England, Precision will Modernise Methods of Relief Printing, The Hand-made Papers of Gaspard Maillol, Scientific Research and the Printing Industry, A Colour Chart for Etching Three-colour Half-tone, The Illustrated Books of Edy Legrand, A Plea for Better Guide Books, Register and Humidity Control, A Theoretical Examination of the Three-colour Process, Youth-Politick in Advertising, Intaglio Litho, Colour Printing for Newspapers by Web Offset Machine, Interleaving, Publicity Photography, A Medium for New Effects in Process Work, Labels as Decoration, The Relationship of Word and Picture, Copper-Bromide or Lead-Ferricyanide? The Chromium Facing and Copper Depositing of Cylinders, Rotary Photogravure, Re-etching Photogravure Cylinders, Lithographic Intaglio Images, Bichromated Albumen in Lithography, Lithographic Half-tone Etching, Metallic Printing Inks—A New Development, A Xylol-free Rotogravure Ink, Is Art Paper Doomed? Recent Lithographic Progress, Making Half-tones with the Erwin Grain Screen, Why not Italic Small Capitals and a Few other Needed Sorts? A Comparison of Printing Processes, Mechanization and Quality, Co-operation in Colour Work, A Simple Fish Glue Formula, Worship of the Past, and the Editor's Note Book. The volume contains numerous illustrations representative of process work during the year 1931, in addition to plates illustrating the articles.